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THE FORCE METHOD ALGORITHM IN THE FORM OF A LOOP RESULTANT METHOD

Abstract. *The object of research is the behavior of statically indeterminate frames under the influence of temperature. The purpose of this work is to suggest a simple algorithm for the analysis of framed structures by using the original idea of the loop resultant method. This basic loop is generated by splitting the given structure into statically indeterminate loops instead of the conventional approach of treating the redundant forces in the whole structure. The current approach allows to simplify the calculation, thanks for using the loop compatibility conditions and by dealing with the primary unknowns for each basic loop. The advantage of this presented approach is in simple structure of a system flexibility matrix: the location of zero and non-zero blocks depend only on the numbering of loops. Different types of flexibility matrices of the element-rods are established; it is shown how to build the compatibility matrix for any loops with or without hinges; and the simple algorithm of the loop resultant method is developed. Some numerical examples are performed to describe the presented algorithm in more detail.*

Keywords: *force method, loop method, flexibility matrix, compatibility matrix, rod systems.*

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АЛГОРИТМ МЕТОДА СИЛ В ФОРМЕ МЕТОДА КОНТУРНЫХ УСИЛИЙ

Аннотация. *Объектом исследования является расчет статически неопределимых стержневых систем при температурных воздействиях. Цель данной работы - предложить простой алгоритм расчета стержневых конструкций с использованием идеи метода контурных усилий. Базовые контуры определяются путем разделения данной конструкции на статически неопределимые контуры вместо традиционного подхода к учету лишних неизвестных метода сил во всей конструкции. Предлагаемый подход позволяет упростить расчет благодаря использованию условий совместности контурных деформаций и автоматическому выбору в качестве лишних неизвестных - усилий для каждого базового контура. Преимущество представленного подхода заключается в простой структуре матрицы системы уравнений – матрицы податливости конструкции: расположение нулевых и ненулевых блоков в которой зависит только от нумерации контуров. Построены в явном виде матрицы податливости элементов-стержней для произвольной системы координат, изложен способ построения матриц совместности деформаций для произвольных контуров, разработан простой алгоритм метода контурных усилий. Приведены некоторые численные примеры для более подробного описания представленного алгоритма.*

Ключевые слова: *метод сил, метод контурных усилий, матрица податливости, матрица совместности, стержневые системы.*

1.Introduction

In architecture and construction, the issue of evaluating the stress-strain state of a building structure and its structural components under static or dynamic effects using an advanced computational model with high reliability of the results is also of great concern.

Thus, the main direction of the development of structural mechanics can be given as the development of new models or the improvement of available computational models that are based on well-known methods such as the force method, the displacement method, the finite element method, or the hybrid numerical method. Over the past two decades, the force method (the flexibility method or the method of consistent deformation) has been successfully applied to static and dynamic analysis of structures.

The force method is commonly used in structural analysis for hand calculations; however it is very difficult to apply it to complex structures in the convential approach by solving a system of equations with so many unknows. Due to the development of matrix algebra and digital electronic computer let to the development of automation of this method, which not only to get rid of the computational difficulties but also to handle these problems efficiently by using the optimal algorithms.

The advantages of the force method can be exploited for structural analysis by different approaches, such as an orthogonal self-stress matrix for space truss structures with cyclic symmetry [1], a structural analysis and optimization algorithm for determining the minimum weight of structures with the truss and beam-type members [2], the formulation using energy principles for design, optimization, and nonlinear analysis [3], the integrated force method, and the extended integrated force method for studying the behavior of structures as trusses and frames [4,5].

Recently, the force method has been expressed as the finite element model, called the finite element force method [6,7] or the finite element integrated force method [8]. In essence, these methods use algebraic functions in combination with the versatility of the classical finite element method as an effective mathematical tool to describe the deformation of structures with the objective of free vibration and buckling analyses.

In structural analysis, two common approaches are used: 1) setting the structural stiffness matrix based on the “finite element displacement” of the nodes in equilibrium conditions; 2) the structural flexibility matrix is constructed by using the “finite element force” of redundant in compatibility conditions. The comparison of these two approaches is shown in the paper [9]. Furthermore, the combination of the two approaches is also found in the paper [10]. Besides, the effectiveness of the second approach is emphasized in papers [11,12]. Proposing the idea of “finite element force” has been around for a long time, and it is used to build efficient models for structural design, which has been developed strongly in recent years through many different paths. Many scientists have continuously perfected the theoretical system based on this idea, and it can be divided into the following main directions:

- The graph-theoretical force method [13];
- The integrated force method [4];
- The generalization of the flexibility method [14,15];
- The hybrid force-based method of the large increment method [16,17].

These works do not contain a sufficiently simple method for constructing flexibility matrices for individual element-rod and the framed structure as a whole. This makes it difficult to create a simple algorithm using the force method, which is convenient for programming. The main goal of this work is to discuss the procedure of the finite element force method, which is based on the loop resultant method [18] in static analysis of framed structures.

2.Materials and Methods

For the approach of the loop resultant method, it is possible to express the internal forces of two nodal cross sections of the element-rod in terms of the resultant parameter at any point A (see Fig. 1).

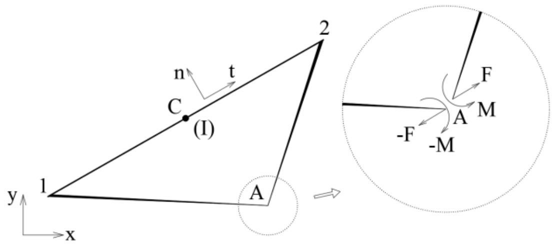

Figure 1 – The first type of the element-rod

The current element-rod L (m) has the tangent and normal vectors $t=[t_x \quad t_y]^T$, $n=[n_x \quad n_y]^T$ respectively and the connection between them according to the expression $t=cn$, here $c=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Point C is called a mid-point of the element-rod.

Rigid consoles are connected to any point A and only this point is used for all element-rods in the rod system. Thus, the behavior of all cross sections of the element-rod is expressed in terms of the resultant force parameters F and M at this point A.

Then we can get three expressions of internal forces corresponding to the position of the cross section (s) of the element-rod:

$$N(s)=F^T t, \tag{1}$$

$$Q(s)=F^T n, \tag{2}$$

$$M(s)=F^T[(-y_{As}) \quad x_{As}]^T+M, \tag{3}$$

where $F=[F_x \quad F_y]^T$ and M - the resultant parameters at point A, x_{As} and y_{As} - the coordinate vector $\overrightarrow{As}$, $(...)^T$ - the transpose of a matrix.

Besides, the strain energy formula can be used conveniently to derive the flexibility matrix of an element-rod, is given by

$$W=\frac{1}{2}\int_0^L\left(\frac{N^2(s)}{k_1}+\frac{M^2(s)}{k_2}+\frac{Q^2(s)}{k_3}\right)ds, \tag{4}$$

where L - the length of the element-rod, $k_1=EA$ - the axial stiffness, $k_2=EI$ - the bending stiffness, $k_3=\kappa GA$ - the shear stiffness.

Substituting the expressions (1), (2) and (3) into the formula (4), we obtain a new expression from the following 17 terms:

$$\begin{aligned} W=\frac{1}{2}\{&\frac{1}{k_1}(F_x^2\int_0^L t_x^2 ds+F_xF_y\int_0^L t_xt_y ds+F_xF_y\int_0^L t_xt_y ds+F_y^2\int_0^L t_y^2 ds) \\ &+\frac{1}{k_2}(F_x^2\int_0^L y_{As}^2 ds+F_xF_y\int_0^L(-x_{As}y_{As})ds+F_xM\int_0^L(-y_{As})ds+F_xF_y\int_0^L(-x_{As}y_{As})ds \\ &+F_y^2\int_0^L x_{As}^2 ds+F_yM\int_0^L x_{As}ds+F_xM\int_0^L(-y_{As})ds+F_yM\int_0^L x_{As}ds+M^2\int_0^L ds) \\ &+\frac{1}{k_3}(F_x^2\int_0^L n_x^2 ds+F_xF_y\int_0^L n_xn_y ds+F_xF_y\int_0^L n_xn_y ds+F_y^2\int_0^L n_y^2 ds)\}. \end{aligned} \tag{5}$$

Then the expression (5) is rewritten in matrix form as follows:

$$W = \frac{1}{2}\Lambda\Theta , \tag{6}$$

where $\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ & \Lambda_{22} & \Lambda_{23} \\ (sym.) & & \Lambda_{33} \end{bmatrix}$, $\Theta = \begin{bmatrix} F_x^2 & F_xF_y & F_xM \\ & F_y^2 & F_yM \\ (sym.) & & M^2 \end{bmatrix}$, $\Lambda_{23} = \frac{1}{k_2}\int_0^L x_{As} ds$,

$\Lambda_{11} = \frac{1}{k_1}\int_0^L t_x^2 ds + \frac{1}{k_3}\int_0^L n_x^2 ds + \frac{1}{k_2}\int_0^L y_{As}^2 ds$, $\Lambda_{33} = \frac{1}{k_2}\int_0^L ds$, $\Lambda_{13} = \frac{1}{k_2}\int_0^L (-y_{As}) ds$,

$\Lambda_{12} = \frac{1}{k_1}\int_0^L t_x t_y ds + \frac{1}{k_3}\int_0^L n_x n_y ds + \frac{1}{k_2}\int_0^L (-x_{As} y_{As}) ds$, $\Lambda_{22} = \frac{1}{k_1}\int_0^L t_y^2 ds + \frac{1}{k_3}\int_0^L n_y^2 ds + \frac{1}{k_2}\int_0^L x_{As}^2 ds$.

The following physical equation can be established [18]:

$$e = \frac{\partial W(\sigma)}{\partial \sigma} . \tag{7}$$

Substituting the expression (6) into the formula (7), we obtain

$$e = \Lambda\sigma , \tag{8}$$

where e, σ - the deformation and strain parameters of the element-rod, Λ is called the flexibility matrix of the element-rod, which depends on the coordinate of point A.

After integrating the expression (5), we obtain a simple form of the flexibility matrix (3x3) with the tangent and normal vectors t, n:

$$\Lambda = \begin{bmatrix} \Lambda_{11}' & \Lambda_{12}' & \Lambda_{13}' \\ & \Lambda_{22}' & \Lambda_{23}' \\ (sym.) & & \Lambda_{33}' \end{bmatrix}, \tag{9}$$

where $\Lambda_{11}' = \frac{L}{k_1}t_x^2 + \frac{L}{k_3}n_x^2 + \frac{L^3}{12k_2}t_y^2 + \frac{L}{k_2}y_{AC}^2$, $\Lambda_{13}' = \frac{L}{k_2}y_{AC}$,

$\Lambda_{12}' = \frac{L}{k_1}t_x t_y + \frac{L}{k_3}n_x n_y + \frac{-L^3}{12k_2}t_x t_y + \frac{-L}{k_2}x_{AC} y_{AC}$, $\Lambda_{22}' = \frac{L}{k_1}t_y^2 + \frac{L}{k_3}n_y^2 + \frac{L^3}{12k_2}t_x^2 + \frac{L}{k_2}x_{AC}^2$,

$\Lambda_{23}' = \frac{-L}{k_2}x_{AC}$, $\Lambda_{33}' = \frac{L}{k_2}$, $x_{AC} = x_A - x_C$, $y_{AC} = y_A - y_C$.

It is easy to see that the flexibility matrix of the first type (3×3) consists of two parts: a constant Λ_C(3x3) and a modified part Λ_A(3x3). Thus, the expression (9) can be rewritten in the following general form:

$$\Lambda_I = \Lambda_C + \Lambda_A = Q\tilde{\Lambda}Q^T + \Lambda_A, \tag{10}$$

where $\tilde{\Lambda} = diag(\frac{L}{k_1}; \frac{L}{k_3} + \frac{L^3}{12k_2}; \frac{L}{k_2})$, $\Lambda_A = \frac{L}{k_2}\begin{bmatrix} y_{AC}^2 & -x_{AC}y_{AC} & -y_{AC} \\ & x_{AC}^2 & x_{AC} \\ (sym.) & & 0 \end{bmatrix}$, and

$Q = \begin{bmatrix} t_x & n_x & 0 \\ t_y & n_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ - the orthogonal matrix (3×3).

In the case two points A and C coincide, the flexibility matrix of the first type without the part Λ_A is obtained by the formula:

$$\Lambda_I = Q\tilde{\Lambda}Q^T = \begin{bmatrix} \Lambda_I^{11} & \Lambda_I^{12} & 0 \\ & \Lambda_I^{22} & 0 \\ (sym.) & & \Lambda_I^{33} \end{bmatrix}, \quad (11)$$

where $\Lambda_I^{11} = \frac{L}{k_1}t_x^2 + \frac{L}{k_3}n_x^2 + \frac{L^3}{12k_2}t_y^2$, $\Lambda_I^{33} = \frac{L}{k_2}$, $\Lambda_I^{12} = \frac{L}{k_1}t_x t_y + \frac{L}{k_3}n_x n_y + \frac{-L^3}{12k_2}t_x t_y$,

$\Lambda_I^{22} = \frac{L}{k_1}t_y^2 + \frac{L}{k_3}n_y^2 + \frac{L^3}{12k_2}t_x^2$.

The remaining forms of the flexibility matrices are obtained for other types of the element-rods using the energy “internal strain” of the first type which can be defined as follows:

$$W_I = \frac{1}{2}\sigma_I^T \Lambda_I \sigma_I, \quad (12)$$

and the components "internal strain" of the first type of the element-rod in matrix form:

$$\sigma_I = [F_x \quad F_y \quad M]^T. \quad (13)$$

The element-rod is hinged at one end and free at the other (see Fig. 2).

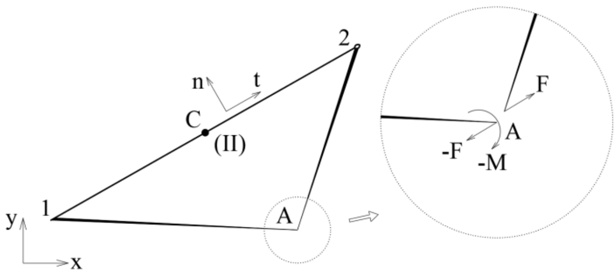

Figure 2 – The second type of the element-rod

The components "internal strain" of the second type of the element-rod can be represented as follows:

$$\sigma_{II} = [F_x \quad F_y]^T. \quad (14)$$

The second type of the element-rod is considered as the case when one hinge is added to the first type of the element-rod, while the bending moment of node 2 is equal to zero. According to the formula (3), we obtain

$$M = -y_{A2}F_x + x_{A2}F_y. \quad (15)$$

Substituting the formula (15) into the expression (13) and we get that the components “internal strain” of the first type of the element-rod do not contain M:

$$\sigma_I = [F_x \quad F_y \quad (-y_{A2}F_x + x_{A2}F_y)]^T. \quad (16)$$

Then we can formulate the relationship between the components "internal strain" of the first and second types of the element-rods using the transformation matrix H1, as shown below:

$$\sigma_I = H_1 \sigma_{II} , \tag{17}$$

where $H_1 = \begin{bmatrix} 1 & 0 & -y_{A2} \\ 0 & 1 & x_{A2} \end{bmatrix}^T$.

Substituting the expression (17) into the formula (12), we obtain

$$W_I = \frac{1}{2}(H_1\sigma_{II})^T \Lambda_I (H_1\sigma_{II}) = \frac{1}{2}\sigma_{II}^T (H_1^T \Lambda_I H_1)\sigma_{II} . \tag{18}$$

Thus, the energy "internal strain" of the second type of the element-rod has the following formula:

$$W_{II} = \frac{1}{2}\sigma_{II}^T \Lambda_{II} \sigma_{II} , \tag{19}$$

where $\Lambda_{II} = H_1^T \Lambda_I H_1$ - the flexibility matrix (2x2) of the second type of the element-rod is shown as below:

$$\Lambda_{II} = \begin{bmatrix} (\frac{L}{k_1}t_x^2 + \frac{L}{k_3}t_y^2 + \frac{L^3}{3k_2}t_y^2) & (\frac{L}{k_1}t_x t_y - \frac{L}{k_3}t_x t_y - \frac{L^3}{3k_2}t_x t_y) \\ (sym.) & (\frac{L}{k_1}t_y^2 + \frac{L}{k_3}t_x^2 + \frac{L^3}{3k_2}t_x^2) \end{bmatrix} . \tag{20}$$

The element-rod with hinged ends (see Fig. 3).

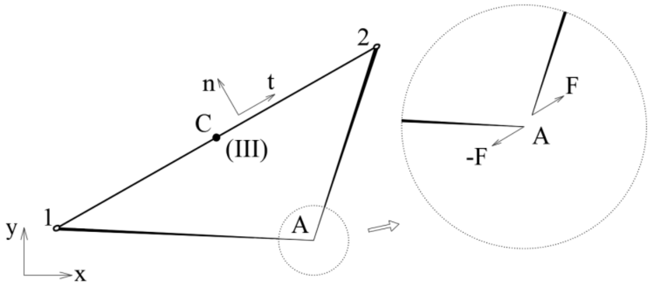

Figure 3 – The third type of the element-rod

The third type of the element-rod is considered as the case when two hinges are added to the first type of the element-rod or as another case when one hinge is added to the second type of the element-rod, while the bending moment of node 1 is equal to zero, i.e., it is calculated by multiplying the expression (2) by the length of this element-rod L: $M_1 = N(s)L = 0$ and we get the following expression:

$$F^T t = 0 . \tag{21}$$

According to the equation (21), we have a connection between the components "internal strain" of the second and third types of the element-rods:

$$H_2 \sigma_{II} = \sigma_{III} , \tag{22}$$

where $H_2 = \begin{bmatrix} t_x & t_y \end{bmatrix}$.

In addition, the energy "internal strain" of the third type of the element-rod can be written as follows:

$$W_{III} = \frac{1}{2}\sigma_{III}^{T}\Lambda_{III}\sigma_{III} . \qquad (23)$$

Substituting the expression (22) into the formula (23), we have

$$W_{III} = \frac{1}{2}(H_2\sigma_{II})^{T}\Lambda_{III}(H_2\sigma_{II}) . \qquad (24)$$

It is possible to consider the formula (24) as the expression of the energy "internal strain" of the second type of the element-rod and rewrite it as follows:

$$W_{II} = \frac{1}{2}\sigma_{II}^{T}(H_2^{T}\Lambda_{III}H_2)\sigma_{II} . \qquad (25)$$

Comparing the expression (25) with the expression (19), we obtain

$$\Lambda_{II} = H_2^{T}\Lambda_{III}H_2 . \qquad (26)$$

Multiplying both parts of the equation (26) by H_2 and H_2^T respectively, we have

$$H_2\Lambda_{II}H_2^{T} = H_2H_2^{T}\Lambda_{III}H_2H_2^{T} . \qquad (27)$$

Thus, the flexibility matrix (1x1) of the third type is

$$\Lambda_{III} = H_2\Lambda_{II}H_2^{T} = \left[\frac{L}{k_1}\right], \qquad (28)$$

where $H_2H_2^T = 1$.

The loop compatibility conditions are applied for the procedure in building an algorithm:

Condition 1. The sum of the degrees of each statically indeterminate loop should be equal to the total degree of the entire statically indeterminate structure.

$$n_{st} = \sum_{i} n_{st}^{1} = n_{st}^{a} + n_{st}^{b} + \ldots + n_{st}^{i} . \qquad (29)$$

Condition 2. Basic loops can have one or several common element-rods and vice versa, but it is necessary to satisfy the condition of independence of each loop, i.e., an individual element-rod must appear at least once in the basic loop.

The algorithm of the loop resultant method can be described in the following steps:

Step 1. Choose the basic loop (see Fig. 4).

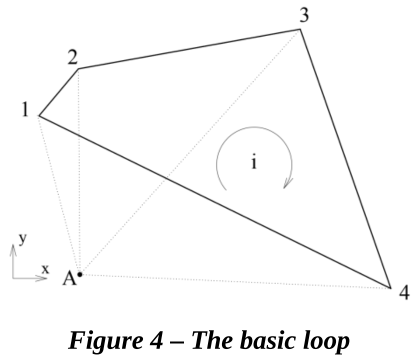

Figure 4 – The basic loop

The compatibility equation of the basic loop i for the deformation should be expressed as below:

$$e_{12} + e_{23} + e_{34} - e_{14} = 0\,. \qquad (30)$$

Step 2. Determine the flexibility matrix of the element-rod Λ_i.

Step 3. Construct the flexibility block diagonal matrix:

$$\Lambda = diag(\Lambda_i)\,. \qquad (31)$$

Step 4. Establish the compatibility matrix B_i.

For the basic loop with two hinges:

$$B_i = [H_2 \quad H_2 \quad H_2 \quad -H_2]\,. \qquad (32)$$

The basic loop with one hinge has

$$B_i = [H_1 \quad H_1 \quad H_1 \quad -H_1]\,, \qquad (33)$$

and the basic loop does not contain the hinge:

$$B_i = [I_3 \quad I_3 \quad I_3 \quad -I_3]\,, \qquad (34)$$

where I_3 - the identity matrix 3×3.

Step 5. Complete the flexibility matrix of the framed structure:

$$L_i = B_i \Lambda B_i^T\,. \qquad (35)$$

Step 6. Solve the system of equations:

$$L_i X_i = -B_i e_o\,, \qquad (36)$$

where X_i - the redundant forces of the basic loops, e_o - the initial deformation of the rod system.

Step 7. Compute the resultant system:

$$\sigma_i = B_i^T X_i\,. \qquad (37)$$

3.Results and Discussion

The results of the loop resultant method are illustrated by numerical examples on different ways of the combination of loops. Here we investigate the following:

First, we consider the triangular and square loops.

Second, we study how to extend the rod system.

The presented algorithm is applied to find bending moments for the following structures.

The element-rod of the structures has a square cross section with the depth h = 0.2 and length L = 2, the Young's modulus $E = 3\times10^{10}$ (Pa), the temperature loads t_1 = 15ºC, t_2 = 5ºC and the coefficient of linear thermal expansion $\alpha = 10^{-5}$ (C^{-1}).

Consider an element-rod subjected to the temperature load. The axial deformation of an element-rod i can be expressed as

$$\Delta_i = L \cdot t \cdot \alpha\,, \qquad (38)$$

where t - average cross section temperature.

The bending deformation of an element-rod i can be written as

$$\theta_i = L \cdot \tau \cdot \alpha\,, \qquad (39)$$

where τ - temperature gradient over the depth.

The temperature load on each element-rod can be expressed in matrix form as below:

$$T = [\Delta_1 \quad \theta_1 \quad \theta_2 \quad \Delta_2 \quad \ldots \quad \ldots \quad \Delta_n \quad \theta_n]. \qquad (40)$$

The basic loops (see Fig. 5).

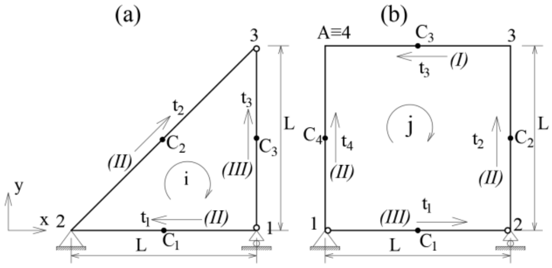

Figure 5 – Various types of loops: (a) the triangular loop, (b) the square loop

Compatibility matrices of basic loops i and j: $B_i = [0 \quad 1 \quad 0 \quad 1 \quad -1]_{(1\times5)}$ and $B_j = [-1 \quad -1 \quad 0 \quad -1 \quad 0 \quad 2 \quad 1 \quad 0]_{(1\times8)}$.

The temperature loads of element-rod 1 (Loop *i*) and element-rod 2 (Loop j): $T_i = [\Delta_1 \quad \theta_1 \quad 0 \quad 0 \quad 0]_{(1\times5)}$ and $T_j = [0 \quad \theta_2 \quad \Delta_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]_{(1\times8)}$.

The results of basic loops are shown in table 1.

Table 1 – The result of bending moments of loops *i* and *j*

No	The bending moment, N·m	
	Loop *i*	Loop *j*
1s	0.00	0.00
1e	+1242.0	0.00
2s	-1242.0	0.00
2e,3s,3e	0.00	+600.0
4s	…	0.00
4e	…	-600.0

Note: The numbers 1, 2, 3 and 4 describe the ordinal number of the element-rod, the value of the bending moment at the start and end nodes of the element-rod denoted by “s” and “e”.

The combination of triangular loops (see Fig. 6).

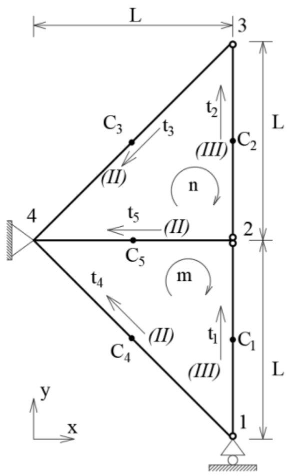

Figure 6 – The plane frame: 5 element-rods

The compatibility matrix: $B = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}_{(2\times8)}$.

The flexibility matrix of the rod system: $L = 10^{-6}\begin{bmatrix} 1.61 & -0.67 \\ (sym.) & 1.61 \end{bmatrix}_{(2\times2)}$.

The temperature load of element-rod 5: $T_5 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \Delta_5 \quad \theta_5]_{(1\times8)}$.

Table 2 – The result of bending moments of the plane frame: 5 element-rods

No	The bending moment, N·m
1s,1e,2s,2e,3s	0.00
3e,4s	-878.679
4e,5s	0.00
5e	+1757.35

The combination of triangular and square loops (see Fig. 7).

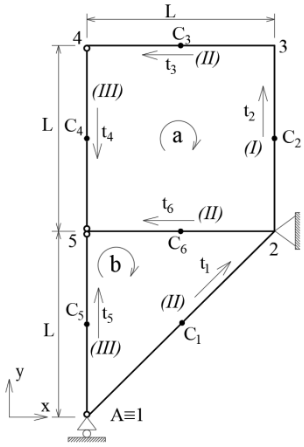

Figure 7 – The plane frame: 6 element-rods

The compatibility matrix: $B = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}_{(2\times11)}$.

The flexibility matrix of the rod system: $L = 10^{-6}\begin{bmatrix} 1.61 & 0.67 \\ (sym.) & 3.33 \end{bmatrix}_{(2\times2)}$.

The temperature load of element-rod 3 (Case 1):

$$T_3 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \Delta_3 \quad \theta_3 \quad 0 \quad 0 \quad 0 \quad 0]_{(1\times11)}.$$

The temperature load of element-rod 6 (Case 2):

$$T_6 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \Delta_6 \quad \theta_6]_{(1\times11)}.$$

The temperature load of element-rods 3 and 6 (Case 3):

$$T_{3,6} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \Delta_3 \quad \theta_3 \quad 0 \quad 0 \quad \Delta_6 \quad \theta_6]_{(1\times11)}.$$

Table 3 – The result of bending moments of the plane frame: 6 element-rods

No.	The bending moment, N·m		
	Case 1	Case 2	Case 3
1s	0.00	0.00	0.00
1e	-270.97	-1083.90	-1354.88
2s,2e,3s	+654.19	-383.218	+270.976
3e,4s,4e,5s,5e	0.00	0.00	0.00
6s	-383.21	+1467.12	+1083.90
6e	0.00	0.00	0.00

The combination of square loops (see Fig. 8).

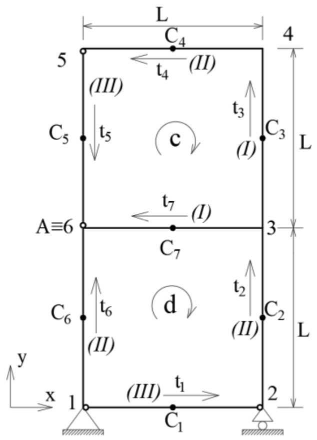

Figure 8 – The plane frame: 7 element-rods

The compatibility matrix:

$$B=\begin{bmatrix}0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1\end{bmatrix}_{(2\times14)}.$$

The flexibility matrix of the rod system: $L=10^{-6}\begin{bmatrix}3.33 & 0.67\\ (sym.) & 3.33\end{bmatrix}_{(2\times2)}$.

The temperature load of element-rod 2 (Case 1):

$$T_2=[0\quad \theta_2\quad \Delta_2\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0]_{(1\times14)}.$$

The temperature load of element-rod 6 (Case 2):

$$T_6=[0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad \theta_6\quad \Delta_6\quad 0\quad 0\quad 0]_{(1\times14)}.$$

The temperature load of element-rods 2 and 4 (Case 3):

$$T_{2,4}=[0\quad \theta_2\quad \Delta_2\quad 0\quad 0\quad 0\quad \Delta_4\quad \theta_4\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0]_{(1\times14)}.$$

The temperature load of element-rods 2, 3 and 6 (Case 4):

$$T_{2,3,6}=[0\quad \theta_2\quad \Delta_2\quad 0\quad 0\quad 0\quad \Delta_3\quad \theta_3\quad 0\quad \theta_6\quad \Delta_6\quad 0\quad 0\quad 0]_{(1\times14)}.$$

Table 4 – The result of bending moments of the plane frame: 7 element-rods

№	The bending moment, N·m			
	Case 1	Case 2	Case 3	Case 4
1s,1e,2s	0.00	0.00	0.00	0.00
2e	+659.34	-659.34	+857.14	+197.80
3s,3e,4s	+197.80	-197.84	+857.14	+659.34
4e,5s,5e	0.00	0.00	0.00	0.00
6s	-659.34	+659.34	-857.14	-197.80
6e	0.00	0.00	0.00	0.00
7s	+461.53	-461.53	0.00	-461.53
7e	+659.34	-659.34	+857.14	+197.80

4.Conclusions

The finite element force method employs a basic idea from the loop resultant method. In this work:

1. the expression of element flexibility matrix is given for any coordinate system;
2. it is shown how to build the compatibility matrix for any loops with or without hinges;

3. an algorithm for constructing the system flexibility matrix is proposed, which makes it easy to program the force method. The examples are studied to illustrate the capability of the loop resultant models for structural design and analysis. In the future, this approach will be extended to carry out dynamic responses and stability problems based on the structure flexibility matrix for the rod systems.

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