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STIFFNESS OF REINFORCED CONCRETE STRUCTURES UNDER BENDING CONSIDERING SHEAR AND AXIAL FORCES (PART 1)

Abstract. *The paper provides a physical and computational model for determining the parameters of limit states of reinforced concrete structures under complex stress state such as bending with effect of axial and shear forces. The forward and backward transitions for determining the stiffness matrix coefficients of reinforced concrete bar structures with inclined and normal cracks have been constructed on the basis of the adopted cross-section discretization scheme and the duality theorem between force and deformation parameters by A.R. Rzhanitsyn. Determination of the stiffness of structures in the zone of inclined cracks was performed on the basis of the model of composite strips into which the zone with inclined cracks is divided. It is assumed a hypothesis about the character of deformation distribution in a complexly stressed reinforced concrete element with inclined cracks. For this model the effective shear modulus has been obtained. It allows to determine the average relative linear and angular strains of concrete and reinforcement at the point adjacent to the shear joint between inclined cracks. Using this model and the experimentally obtained value of the relative shear in the inclined crack, the dowel forces in the reinforcing bar crossed by the inclined crack were determined. The use of the obtained analytical dependences in the practice of designing reinforced concrete structures allows to clarify significantly the definition of displacements and width of opening of inclined and normal cracks, as well as to bring the calculation and physical model based on experimental data as close as possible.*

Keywords: *reinforced concrete, stiffness, physical model, computational model, inclined crack, shear, dowel effect, composite bar.*

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ЖЕСТКОСТЬ ЖЕЛЕЗОБЕТОННЫХ КОНСТРУКЦИЙ ПРИ ИЗГИБЕ С УЧЁТОМ ПОПЕРЕЧНОЙ И ПРОДОЛЬНОЙ СИЛ (ЧАСТЬ 1)

Аннотация. *Приведены физическая и расчётная модель для определения параметров предельных состояний железобетонных конструкций при сложном напряжённом состоянии-изгибе с продольной и поперечными силами. На основе принятой схемы дискретизации поперечного сечения и теоремы двойственности между силовыми и деформационными параметрами А.Р. Ржаницына построены прямой и обратный переход для определения коэффициентов матрицы жёсткости железобетонных стержневых конструкций с наклонными и нормальными трещинами. Определение жёсткости конструкций в зоне наклонных трещин выполнено на основе модели составных полосок, на которые разбивается зона с наклонными трещинами. При этом принята гипотеза о характере распределения деформаций в сложно напряжённом железобетонном элементе с наклонными трещинами. Для этой модели получен условный модуль сдвига позволяющий определять средние относительные линейные и угловые деформации бетоны и арматуры в точке прилегающей к шву сдвига между наклонными трещинами. На основе этой модели и с использованием экспериментально полученного значения сдвига в наклонной трещине определены нагельные усилия в* *арматурном стержне, пересекаемом наклонной трещиной. Использование полученных аналитических зависимостей в практике проектирования железобетонных конструкций позволяет не только существенно уточнить определение перемещений и ширины раскрытия наклонных и нормальных трещин, но и максимально сблизить расчётную и физическую модель, базирующуюся на экспериментальных данных.*

Ключевые слова: *железобетон, жесткость, физическая модель, расчётная модель, наклонная трещина, сдвиг, нагельный эффект, составной стержень.*

Intbaruction

The development of research on the creation of new calculation models with complex stress states [1-4, 6, 7] is associated with an increase in the accuracy and reliability of calculation reinforced concrete structures buildings and constructions. The main factor in the creation of effective building structures is the emergence of new technologies at the convergence of physical phenomena, theory and practice of calculation of reinforced concrete [5-9, 19, 20].

The main purpose of long-term experimental and theoretical research of inclined cracks for bending elements, considering shear and axial forces, was to assess the crack resistance and strength of reinforced concrete structures [10-22].

However, there are few researches [16-21] to determine the stiffness of structures considering inclined cracks, including intersecting cracks. This article contains the developed model for estimating the stiffness of reinforced concrete structures under bending taking into account effect of shear and axial forces with inclined cracks using single composite strips in the block - in the wedge and arch between inclined cracks. In addition, the approximation of rectangular cross-sections using small squares in the stiffness matrix elements is also considered.

Based on the analysis of the research of N.I. Karpenko and S.N. Karpenko [23-25], in the axial tensile reinforcement the "dowel" forces Q_s and drift in the crack Δ_s are obtained as a function of the opening width and strains in concrete to the cosine of the crack slope angle (θ). The experimental value " dowel " forces $Q_{s,\exp}$ and experimental drift in a crack dependence for the connection between drift ($\Delta_{crc,exp}$) and shear span (a/h_0) are determined, also experimental relationship between an anchoring zone length (and reinforcement strains [10] ($\varepsilon_s \cdot m_{s,3}$).

The composite strip calculation model and approximation model for rectangular cross-sections using small squares in the stiffness matrix elements

A new effect of the theory of reinforced concrete is established, the concept of conditional modulus of joint (ξ_m) has been intbaruced based on the hypotheses presented in the papers [16-18, 22]. The conditional modulus of joint is defined in a single compound for the shear zone of the joint for the difference of average relative linear and angular $\gamma_{zx,stitch,s,i}$ strains of reciprocal displacements of the concrete (or reinforcement). This allows for reinforced concrete to reduce A.R. Rzhanitsyn's system of differential equations by an order of magnitude and to obtain the total angular strains $\gamma_{zx,stitch,sum,i}$ of concrete in each shear joint and “dowel” effect of reinforcement deformation. The physical characteristics stiffness matrix compressed zone of concrete and the working reinforcement are derived in an equation system of static, deformation and physical equations.

The bending element stiffness considering axial and shear forces is calculated by approximation with small squares of the cross-sections of the structure with lateral inclined and normal cracks (figure 1). Bending moment $M_{bend,ij}$, axial force N_{ij} and shear force Q_{ij} diagrams are plotted for a number of sections 1 to 6 in the crack area. The stiffness characteristics of the sections with cracks D_{ij} (i, j - 1, 2, 3) for equations from [19-21] are obtained in the system of reinforced concrete blocks that is the wedges and arch in the area between inclined cracks ($l_{crc,i}$).

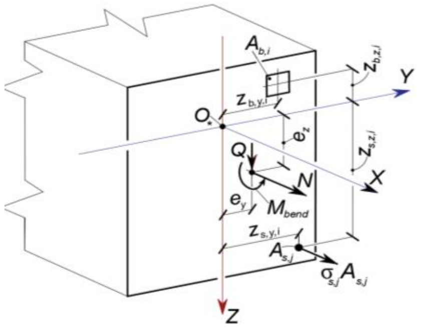

Figure 1 - Small-square approximation scheme in the tensile and compressed zones of a bending element cross-section, taking into account the action of shear and axial forces and the presence of inclined and normal cracks

To solve the problem, we perform a direct transition with known forces M_x, N, Q, but unknown deformations is performed $\frac{1}{r_x}$, ε_0 and Δ_Q:

$$\frac{1}{r_x} = D_{11,*} \cdot M_x + D_{13,*} \cdot N + D_{14,*} \cdot Q \quad ; \qquad (1)$$

$$\varepsilon_0 = D_{31,*} \cdot M_x + D_{33,*} \cdot N + D_{34,*} \cdot Q \quad ; \qquad (2)$$

$$\Delta_Q = D_{41,*} \cdot M_x + D_{43,*} \cdot N + D_{44,*} \cdot Q \quad ; \qquad (3)$$

write down the expression for M_x from the equation (1):

$$M_x = \frac{1}{D_{11,*}}\left(\frac{1}{r_x} - D_{13,*} \cdot N - D_{14,*} \cdot Q\right) . \qquad (4)$$

Find the values N from the equation (2):

$$N = \frac{1}{D_{33,*}}\left(\varepsilon_0 - D_{31,*} \cdot M_x - D_{34,*} \cdot Q\right) . \qquad (5)$$

Find the values Q from the equation (3):

$$Q = \frac{1}{D_{44,*}}\left(\Delta_Q - D_{41,*} \cdot M_x - D_{43,*} \cdot N\right) ; \qquad (6)$$

We can then write down a system of equations:

$$\begin{cases} M_x = \dfrac{\frac{1}{r_x} - D_{13,*} \cdot N - D_{14,*} \cdot Q}{D_{11,*}} \\ N = \dfrac{\varepsilon_0 - D_{31,*} \cdot M_x - D_{34,*} \cdot Q}{D_{33,*}} \\ Q = \dfrac{\Delta_Q - D_{41,*} \cdot M_x - D_{43,*} \cdot N}{D_{44,*}} \end{cases} ; \qquad (7)$$

Substitute the value of *N* from the second equation of this system into the first equation of system (7):

$$M_x = \frac{1}{r_x} \cdot \frac{D_{33,*}}{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}} + \varepsilon_0 \frac{-D_{13,*}}{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}} + Q\frac{D_{13,*} \cdot D_{34,*} - D_{33,*} \cdot D_{14,*}}{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}}; \quad (8)$$

Then substitute the value of M_x from the first equation of this system into the second equation of system (7)

$$N = \varepsilon_0 \cdot \frac{D_{11,*}}{D_{33,*}D_{11,*} - D_{31,*}D_{13,*}} + \frac{1}{r_x} \cdot \frac{-D_{31,*}}{D_{33,*}D_{11,*} - D_{31,*}D_{13,*}} + Q \cdot \frac{D_{31,*}D_{14,*} - D_{11,*}D_{34,*}}{D_{33,*}D_{11,*} - D_{31,*}D_{13,*}}; \quad (9)$$

Then let's substitute the obtained value M_x and from equation (8) and (9) into the third equation of the system:

$$Q = D_{41,**} \cdot \frac{1}{r_x} + D_{43,**} \cdot \varepsilon_0 + D_{44,**} \cdot \Delta_Q, \quad (10)$$

here the relevant coefficients are of the form D:

$$D_{41,**} = \frac{-D_{41,*} \cdot D_{33,*} + D_{43,*} \cdot D_{31,*}}{D_{44,*}D_{11,*}D_{33,*} - D_{44,*}D_{13,*} \cdot D_{31,*} + D_{41,*} \cdot D_{13,*} \cdot D_{34,*} - D_{41,*} \cdot D_{33,*} \cdot D_{14,*} + D_{43,*} \cdot D_{31,*}D_{14,*} - D_{43,*} \cdot D_{11,*}D_{34,*}}; \quad (11)$$

$$D_{43,**} = \frac{D_{41,*} \cdot D_{13,*} - D_{43,*} \cdot D_{11,*}}{D_{44,*}D_{11,*}D_{33,*} - D_{44,*}D_{13,*} \cdot D_{31,*} + D_{41,*} \cdot D_{13,*} \cdot D_{34,*} - D_{41,*} \cdot D_{33,*} \cdot D_{14,*} + D_{43,*} \cdot D_{31,*}D_{14,*} - D_{43,*} \cdot D_{11,*}D_{34,*}}; \quad (12)$$

$$D_{44,**} = \frac{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}}{D_{44,*}D_{11,*}D_{33,*} - D_{44,*}D_{13,*} \cdot D_{31,*} + D_{41,*} \cdot D_{13,*} \cdot D_{34,*} - D_{41,*} \cdot D_{33,*} \cdot D_{14,*} + D_{43,*} \cdot D_{31,*}D_{14,*} - D_{43,*} \cdot D_{11,*}D_{34,*}}; \quad (13)$$

Substitute the value Q from equation (10) into equation (8) we obtain:

$$M_x = D_{11,**} \cdot \frac{1}{r_x} + D_{13,**} \cdot \varepsilon_0 + D_{14,**} \cdot \Delta_Q. \quad (14)$$

Here the relevant coefficients take the form of *D*:

$$D_{11,**} = \frac{D_{33,*} + D_{41,**} \cdot D_{13,*} \cdot D_{34,*} - D_{41,**} \cdot D_{33,*} \cdot D_{14,*}}{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}}; \quad (15)$$

$$D_{13,**} = \frac{-D_{13,*} + D_{43,**}D_{13,*} \cdot D_{34,*} - D_{43,**}D_{33,*} \cdot D_{14,*}}{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}}; \quad (16)$$

$$D_{14,**} = \frac{D_{44,**} \cdot D_{13,*} \cdot D_{34,*} - D_{44,**} \cdot D_{33,*} \cdot D_{14,*}}{D_{11,*}D_{33,*} - D_{13,*} \cdot D_{31,*}}; \quad (17)$$

Similarly for *N* we obtain:

$$N = D_{31,**} \cdot \frac{1}{r_x} + D_{33,**} \cdot \varepsilon_0 + D_{34,**} \cdot \Delta_Q; \quad (18)$$

Here the relevant coefficients take the form of *D*:

$$D_{31,**} = \frac{-D_{31,*} + D_{41,**} \cdot D_{31,*}D_{14,*} - D_{41,**} \cdot D_{11,*}D_{34,*}}{D_{33,*}D_{11,*} - D_{31,*}D_{13,*}}; \quad (19)$$

$$D_{33,**} = \frac{D_{11,*} + D_{43,**} \cdot D_{31,*}D_{14,*} - D_{43,**} \cdot D_{11,*}D_{34,*}}{D_{33,*}D_{11,*} - D_{31,*}D_{13,*}}; \quad (20)$$

$$D_{34,**} = \frac{D_{44,**} \cdot D_{31,*}D_{14,*} - D_{44,**} \cdot D_{11,*}D_{34,*}}{D_{33,*}D_{11,*} - D_{31,*}D_{13,*}}; \quad (21)$$

As a result, we can write the matrix D:

$$D = \begin{bmatrix} D_{11,**} & D_{13,**} & D_{14,**} \\ D_{31,**} & D_{33,**} & D_{34,**} \\ D_{41,**} & D_{43,**} & D_{44,**} \end{bmatrix}, \tag{22}$$

where $D_{11,**}, D_{13,**}, D_{14,**}, D_{31,**}, D_{33,**}, D_{34,**}, D_{41,**}, D_{43,**}, D_{44,**}$, are defined from equation system:

$$\begin{cases} M_x = D_{11,**} \cdot \dfrac{1}{r_x} + D_{13,**} \cdot \varepsilon_0 + D_{14,**} \cdot \Delta_Q; \\ N = D_{31,**} \cdot \dfrac{1}{r_x} + D_{33,**} \cdot \varepsilon_0 + D_{34,**} \cdot \Delta_Q; \\ Q = D_{41,**} \cdot \dfrac{1}{r_x} + D_{43,**} \cdot \varepsilon_0 + D_{44,**} \cdot \Delta_Q. \end{cases}$$

$D_{11,**}$ is the second-level coefficient in the matrix elements in equation (22), which is written in the form:

$$D_{11,**} = \frac{D_{33,*} + D_{41,**} \cdot D_{13,*} \cdot D_{34,*} - D_{41,**} \cdot D_{33,*} \cdot D_{14,*}}{D_{11,*} D_{33,*} - D_{13,*} \cdot D_{31,*}}, \tag{23}$$

$D_{11,*}$ and $D_{41,*}$ $(D_{12,*} = 0)$ are the first-level coefficient in equations (19-21). The coefficients are also obtained: $D_{11,*}$ - $D_{44,*}$.

The coefficient $D_{41,*}$ in the matrix elements at the first level is determined from equation:

$$D_{41,*} = \frac{-D_{14} \cdot D_{33} + D_{34} \cdot D_{13}}{D_{44} D_{11} D_{33} - D_{44} D_{13} \cdot D_{13} + 2D_{14} \cdot D_{13} \cdot D_{34} - D_{14} \cdot D_{14} \cdot D_{33} - D_{34} \cdot D_{34} D_{11}}. \tag{24}$$

Here D_{11} - D_{14} are the coefficients in the matrix elements, at the first level are determined according to the formulas SP 63.13330.2018.

Now perform the reverse transition when the curvatures and strains in the element cross-section $\frac{1}{r_x}$, ε_0 and ΔQ are known, but the forces M_x, N, Q are unknown:

$$M_x = D_{11} \cdot \frac{1}{r_x} + D_{13} \cdot \varepsilon_0 + D_{14} \cdot \Delta_Q; \tag{25}$$

$$N = D_{13} \cdot \frac{1}{r_x} + D_{33} \cdot \varepsilon_0 + D_{34} \cdot \Delta_Q; \tag{26}$$

$$Q = D_{14} \cdot \frac{1}{r_x} + D_{34} \cdot \varepsilon_0 + D_{44} \cdot \Delta_Q. \tag{27}$$

Get the curvature values $\frac{1}{r_x}$ from the expression (25):

$$\frac{1}{r_x} = \frac{M_x - D_{13} \cdot \varepsilon_0 - D_{14} \cdot \Delta Q}{D_{11}}, \tag{28}$$

The values of axial strains ε_0 and drifts ΔQ we obtain from expressions (26) and (27) respectively:

$$\varepsilon_0 = \frac{N \cdot \; -D_{13} \cdot \; \frac{1}{r_x} - D_{34} \cdot \; \Delta Q}{D_{33}};\qquad (29)$$

$$\Delta Q = \frac{Q - D_{14} \cdot \; \frac{1}{r_x} - D_{34} \cdot \; \varepsilon_0}{D_{44}}.\qquad (30)$$

As a result, we obtain a system of equations similar to (22):

$$\begin{cases} \frac{1}{r_x} = \frac{M_x - D_{13} \cdot \varepsilon_0 - D_{14} \cdot \Delta Q}{D_{11}}; \\ \varepsilon_0 = \frac{N - D_{13} \cdot \frac{1}{r_x} - D_{34} \cdot \Delta Q}{D_{33}}; \\ \Delta Q = \frac{Q - D_{14} \cdot \frac{1}{r_x} - D_{34} \cdot \varepsilon_0}{D_{44}}. \end{cases}\qquad (31)$$

In the first equation of the system (31), substitute the strain value from the second equation of the system:

$$\frac{1}{r_x} = \frac{D_{33}}{D_{11}D_{33} - D_{13} \cdot \; D_{13}} \cdot \; M_x + \frac{-D_{13}}{D_{11}D_{33} - D_{13} \cdot \; D_{13}} \cdot \; N + \frac{D_{13} \cdot \; D_{34} - D_{14} \cdot \; D_{33}}{D_{11}D_{33} - D_{13} \cdot \; D_{13}} \cdot \; \Delta Q.\qquad (32)$$

Then substitute the curvature values $\frac{1}{r_x}$ from the first equation of the system into the second equation of the system (31):

$$\varepsilon_0 = \frac{D_{11}}{D_{33}D_{11} - D_{13} \cdot \; D_{13}} \cdot \; N + \frac{-D_{13}}{D_{33}D_{11} - D_{13} \cdot \; D_{13}} \cdot \; M_x + \frac{D_{13} \cdot \; D_{14} - D_{34}D_{11}}{D_{33}D_{11} - D_{13} \cdot \; D_{13}} \cdot \; \Delta Q.\qquad (33)$$

Now substitute the curvature value $\frac{1}{r_x}$ and axial strains ε_0 obtained from equations (32) and (33) into the third equation of the system (31). The result is as follows:

$$\Delta Q = D_{41,*} \cdot \; M_x + D_{43,*} \cdot \; N + D_{44,*} \cdot \; Q.\qquad (34)$$

Here the coefficients *D* are of the form:

$$D_{41,*} = \frac{-D_{14} \cdot \; D_{33} + D_{34} \cdot \; D_{13}}{D_{44}D_{11}D_{33} - D_{44}D_{13} \cdot \; D_{13} + 2D_{14} \cdot \; D_{13} \cdot \; D_{34} - D_{14} \cdot \; D_{14} \cdot \; D_{33} - D_{34} \cdot \; D_{34}D_{11}};\qquad (35)$$

$$D_{44,*} = \frac{D_{11}D_{33} - D_{13} \cdot \; D_{13}}{D_{44}D_{11}D_{33} - D_{44}D_{13} \cdot \; D_{13} + 2D_{14} \cdot \; D_{13} \cdot \; D_{34} - D_{14} \cdot \; D_{14} \cdot \; D_{33} - D_{34} \cdot \; D_{34}D_{11}};\qquad (36)$$

$$D_{43,*} = \frac{D_{14} \cdot \; D_{13} - D_{34} \cdot \; D_{11}}{D_{44}D_{11}D_{33} - D_{44}D_{13} \cdot \; D_{13} + 2D_{14} \cdot \; D_{13} \cdot \; D_{34} - D_{14} \cdot \; D_{14} \cdot \; D_{33} - D_{34} \cdot \; D_{34}D_{11}}.\qquad (37)$$

And then substitute the value of ΔQ obtained from equation (34) into the expression (32) and get as a result:

$$\frac{1}{r_x} = D_{11,*} \cdot \; M_x + D_{13,*} \cdot \; N + D_{14,*} \cdot \; Q;\qquad (38)$$

Here the coefficients *D* are of the form:

$$D_{11,*} = \frac{D_{33} + D_{41,*} \cdot\ D_{13} \cdot\ D_{34} - D_{41,*} \cdot\ D_{14} \cdot\ D_{33}}{D_{11}D_{33} - D_{13} \cdot\ D_{13}}; \tag{39}$$

$$D_{13,*} = \frac{-D_{13} + D_{13} \cdot\ D_{34} \cdot\ D_{43,*} \cdot\ -D_{14} \cdot\ D_{33} \cdot\ D_{43,*}}{D_{11}D_{33} - D_{13} \cdot\ D_{13}}; \tag{40}$$

$$D_{14,*} = \frac{D_{13} \cdot\ D_{34} \cdot\ D_{44,*} - D_{14} \cdot\ D_{33} \cdot\ D_{44,*}}{D_{11}D_{33} - D_{13} \cdot\ D_{13}}; \tag{41}$$

After that substitute the value of ΔQ from equation (34) into the expression (33) and we obtain:

$$\varepsilon_0 = D_{31},*\cdot M_x + D_{33},*\cdot N + D_{34},*\cdot Q. \tag{42}$$

Here the coefficients D are of the form:

$$D_{31},* = \frac{D_{13} \cdot D_{14} \cdot D_{41,*} - D_{34}D_{11} \cdot D_{41,*} - D_{13}}{D_{33}D_{11} - D_{13} \cdot D_{13}}; \tag{43}$$

$$D_{33},* = \frac{D_{11} + D_{13} \cdot D_{14} \cdot D_{43,*} - D_{34}D_{11} \cdot D_{43,*}}{D_{33}D_{11} - D_{13} \cdot D_{13}}; \tag{44}$$

$$D_{34},* = \frac{D_{13} \cdot D_{14} \cdot D_{44,*} - D_{34}D_{11} \cdot D_{44,*}}{D_{33}D_{11} - D_{13} \cdot D_{13}}. \tag{45}$$

As a result, we obtain the following system for obtaining the coefficients of matrix D:

$$\begin{cases} \dfrac{1}{r_x} = D_{11,*} \cdot M_x + D_{13,*} \cdot N + D_{14,*} \cdot Q; \\ \varepsilon_0 = D_{31,*} \cdot M_x + D_{33,*} \cdot N + D_{34,*} \cdot Q; \\ \Delta Q = D_{41,*} \cdot M_x + D_{43,*} \cdot N + D_{44,*} \cdot Q. \end{cases}, \tag{46}$$

here

$$D_{13,*} = \frac{D_{13} \cdot\ D_{34} \cdot\ D_{43,*} \cdot\ -D_{14} \cdot\ D_{33} \cdot\ D_{43,*} - D_{13}}{D_{11}D_{33} - D_{13} \cdot\ D_{13}}; \tag{47}$$

$$D_{14,*} = \frac{D_{13} \cdot\ D_{34} \cdot\ D_{44,*} - D_{14} \cdot\ D_{33} \cdot\ D_{44,*}}{D_{11}D_{33} - D_{13} \cdot\ D_{13}}; \tag{48}$$

$$D_{31},* = \frac{D_{13} \cdot D_{14} \cdot D_{41,*} - D_{34}D_{11} \cdot D_{41,*} - D_{13}}{D_{33}D_{11} - D_{13} \cdot D_{13}}; \tag{49}$$

$$D_{33},* = \frac{D_{11} + D_{13} \cdot D_{14} \cdot D_{43,*} - D_{34}D_{11} \cdot D_{43,*}}{D_{33}D_{11} - D_{13} \cdot D_{13}}; \tag{50}$$

$$D_{34},* = \frac{D_{13} \cdot D_{14} \cdot D_{44,*} - D_{34}D_{11} \cdot D_{44,*}}{D_{33}D_{11} - D_{13} \cdot D_{13}}; \tag{51}$$

$$D_{41,*} = \frac{-D_{14} \cdot\ D_{33} + D_{34} \cdot\ D_{13}}{D_{44}D_{11}D_{33} - D_{44}D_{13} \cdot\ D_{13} + 2D_{14} \cdot\ D_{13} \cdot\ D_{34} - D_{14} \cdot\ D_{14} \cdot\ D_{33} - D_{34} \cdot\ D_{34}D_{11}}; \tag{52}$$

$$D_{43,*} = \frac{D_{14}\cdot D_{13} - D_{34}\cdot D_{11}}{D_{44}D_{11}D_{33} - D_{44}D_{13}\cdot D_{13} + 2D_{14}\cdot D_{13}\cdot D_{34} - D_{14}\cdot D_{14}\cdot D_{33} - D_{34}\cdot D_{34}D_{11}}; \quad (53)$$

$$D_{44,*} = \frac{D_{11}D_{33} - D_{13}\cdot D_{13}}{D_{44}D_{11}D_{33} - D_{44}D_{13}\cdot D_{13} + 2D_{14}\cdot D_{13}\cdot D_{34} - D_{14}\cdot D_{14}\cdot D_{33} - D_{34}\cdot D_{34}D_{11}}. \quad (54)$$

Thus, the matrix elements $D_{p,q}$ are determined by the given transformations. And there is also a ***direct transition 1*** from the internal forces M_x, N, Q in the section of the structure to deformations $\frac{1}{r_x}$, ε_0, Δ_Q.

The mathematical levels obtained are from the first to the second level, respectively.

Equations of a reinforced concrete composite bar and formulas for determining its stiffness parameters

Write down the dependences for the axial strains $\varepsilon_{0,x,j,i}$, curvatures $\frac{1}{r_{y,j,i}}$ and shear drifts $\Delta_{Q,\sum}$ using the proposed discretization scheme of the section by small squares.

Axial strain $\varepsilon_{b,0,x,j,i}$ due to the bending of the neutral axis (N.A.) $\varepsilon_{b,0,x,j,i*}$ obtained in [10, 16-21] has the form:

$$\varepsilon_{b,0,x,j,i} = \varepsilon_{b,0,x,j,i*} + \frac{a_{crc,0}}{l_{crc,0}}. \quad (55)$$

Here $\Delta\varepsilon_{b,0,x,j,i}$ is the internal additional axial strain caused by crack opening a_{crc} from concrete spalling $\Delta\varepsilon_{b,0,x,j,i} = \frac{a_{crc,0}}{l_{crc,0}}$ or reinforcement strains $\Delta\varepsilon_{s,0,x,j,i} = \frac{a_{crc,0}\cdot k_r}{l_{crc,0}}$;

internal additions of axial strain $\Delta\varepsilon_{b,0,x,j,i}$ and $\Delta\varepsilon_{s,0,x,j,i}$ fibers, located in the neutral axis (i.e., point O*) of concrete or reinforcement.

Determine the curvature of the reinforced concrete element $\frac{1}{r_{y,j,i}}$ using the equation (56):

$$\frac{1}{r_{y,j,i}} = \frac{\varepsilon_{n,up,i}}{z_{n,up,i} - z_{n,b,A,0,i}} = \frac{\varepsilon_{n,up,i} + \varepsilon_{n,d,i}}{\Delta h_i} \pm \Delta_{j,i}\left(\frac{1}{r_x}\right) = \frac{\varepsilon_{n,up,i} + \varepsilon_{n,d,i}}{\Delta h_i} \pm \frac{\left(\frac{a_{crc}\cdot k_r}{l_{crc}}\right)_{n,up,i} + \left(\frac{a_{crc}\cdot k_r}{l_{crc}}\right)_{n,d,i}}{\Delta h_i}. \quad (56)$$

Here $\varepsilon_{n,up,1,i}$ and $\varepsilon_{n,d,1,i}$ are the strains in the upper and lower areas of one small square; Δh_i is the small square height; $\pm\Delta_{j,i}\left(\frac{1}{r_x}\right)$ is the internal addition from the crack opening a_{crc} for concrete $\Delta\varepsilon_{b,up,x,j,i} = a_{crc}\cdot k_r / l_{crc}$ or for reinforcement $\Delta\varepsilon_{s,d,x,j,i} = a_{crc} / l_{crc}$.

Shear drift from shear force Q is:

$$\Delta_{Q,b,\sum} = \Delta_{Q,j,i} + \Delta_{\partial on,b} = \frac{Q_{j,i}}{G_b(\lambda)\cdot A_{b,j,i}}\cdot\eta_{Q,b} + \Delta_{add,zx,b}(k_r); \quad (57)$$

$$\Delta_{Q,s,\sum} = \frac{Q_{j,i}}{G_s(\lambda)\cdot\frac{A_{s,j,i}}{\mu_s}}\cdot\eta_{Q,s} + \Delta_{add,zx,s}(k_r) \quad . \tag{58}$$

Here $\Delta_{Q,\sum}$ is the total shear drifts from shear force $\Delta_{Q,j,i}$ as for a solid body, defined by structural mechanics methods; $\Delta_{add,zx,b}(k_r)$ is the internal (from crack opening) addition of drift (i.e., $\Delta_{crc,zx}$).

Then, determine ***the drift*** Δ_Q ***from the shear force*** in the inclined section. Then the stiffness of the section in the zone of inclined cracks will be obtained through intersecting cracks using the single composite strip model and the block that is wedge and arch $l_{crc,i}$ model in the area of the curved axis for internal forces (figure 2).

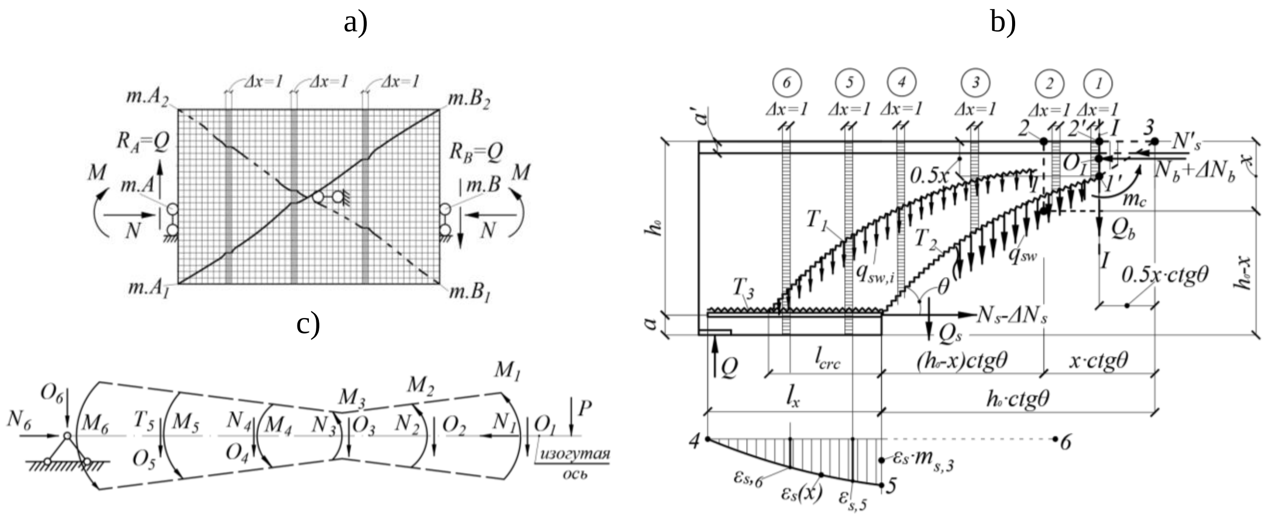

Figure 2 - Inclined (crossed) cracks and shear single composite strips (a); block - wedge and arches $l_{crc,i}$ ***(b), curved axis for internal forces (c)***

The proposed hypothesis and model of composite bar [16-22] allows the system of differential equations of Prof. A.R. Rzhanitsyn to be reduced by an order of magnitude in the presence of cracks arising at an arbitrary point of shear joint.

Following [16-22] for the reinforced concrete composite bar model in question, it can be written:

$$k_{i,m}\cdot\gamma_{b,m} = \varepsilon_{qm}; \tag{59}$$

$$\tau_q = k_{i,m}\cdot\gamma_{b,m}\cdot\xi_m = \varepsilon_{qm}\xi_m \tag{60}$$

$$\tau' = \frac{T''}{\xi_\Delta}; \tag{61}$$

$$\tau' = \frac{T'}{\xi_m} = \frac{T'}{\xi_m} = \gamma T + \Delta \quad ; \tag{62}$$

$$\begin{cases} \frac{T'_1}{\xi_{m,1}} - \Delta_{11}T_1 - \Delta_{12}T_1 - ... - \Delta_{in}T_n = \Delta_{10}; \\ \frac{T'_2}{\xi_{m,2}} - \Delta_{21}T_1 - \Delta_{22}T_2 - ... - \Delta_{2n}T_n = \Delta_{20}; \\ \frac{T'_n}{\xi_{m,n}} - \Delta_{ni}T_i - \Delta_{n2}T_2 - ... - \Delta_{nn}T_n = \Delta_{n0}. \end{cases} \quad (63)$$

For calculated element with cracks:

$$\Delta = -\frac{N_{0.1}}{(E_{b,1}A_{b,1})_{ekv}} + \frac{N_{0.2}}{(E_{b,2}A_{b,2})_{ekv}} - \frac{f(x_{crc})}{r_y}; \quad (64)$$

$$\lambda = \sqrt{\xi\gamma} = \sqrt{\xi\left[\frac{1}{(E_{b,1}A_{b,1})_{ekv}} + \frac{1}{(E_{b,2}A_{b,2})_{ekv}} + \frac{f^2(x_{crc})}{M \times r_y}\right]}; \quad (65)$$

For (63) T_1, T_2, T_n are the shear forces in the 1st, 2nd, …, *n*th bars accumulated along the length of the bar to the section; $\xi_{m,1}$, $\xi_{m,2}$, $\xi_{m,n}$ are modulus in a single shear band of the joint, Δ is the drift along a given direction; ε_{qm}, $\gamma_{b,m}$ are the difference in the average linear and angular strains of concrete and reinforcement in the joint point adjacent to the joint between adjacent cracks; $(E_{b,1}A_{b,1})_{ekv}$, $(E_{b,2}A_{b,2})_{ekv}$ are the equivalent sectional stiffnesses [10, 16-22].

The experimental dependence of the crack drift $\Delta_{crc,exp}(a/h_0)$ is plotted from experimental data [10] graphically (figure 3):

$$\Delta_{crc,exp} = e^{-\lambda\left(\frac{a}{h_0}+A\right)} + C \quad . \quad (66)$$

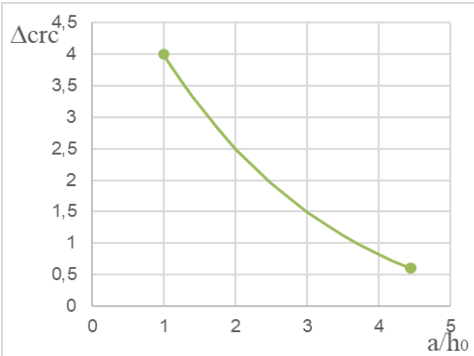

Figure 3 - Dependency graph $\Delta_{crs} - \frac{a}{h_0}$

The above exponent has the following characteristic points:

p.1 - $\Delta_{\text{crc},1} = 4\ mm$, $a_1/h_0 = 1$ mm; p.2 - $\Delta_{\text{crc},2} = 1.5\ mm$, $a_2/h_0 = 3$; p.3 - $\Delta_{crc,3} = 1$ mm, $a_3/h_0 = 3.7$ mm.

From equation (66) we obtain: $4 = e^{-\lambda(1+A)} + C$; $1{,}5 = e^{-\lambda(3+A)} + C$; $1 = e^{-\lambda(3{,}7+A)} + C$.

The result is as follows: $\lambda = 0{,}397512$; $A = -4{,}816204$; $C = -0{,}55847$.

After the constants determination dependence (66) takes the form:

$$\Delta_{crc} = e^{-0{,}397512\left(\frac{a}{h_0}-4{,}816204\right)} - 0{,}55847 \quad . \quad (67)$$

It is important to definite the dependence of the strains of the reinforcement ε_S in the anchoring zone (Fig. 2b) in the form of:

$$\varepsilon_s \cdot m_{s,3} = A \cdot l_x{}^2 + B \cdot l_x + C . \quad (68)$$

Here $\frac{l_x}{l_{an}} = m_{s,3}$ is the coefficient and the parameters for the anchoring zone: $A = \frac{\varepsilon_s \cdot m_{s,3}}{l_x{}^2 - 2l_x}$; $B = -\frac{\varepsilon_s \cdot m_{s,3}}{l_x{}^2 - 2l_x} 2l_x$; $C = 0$.

Then we obtain:

$$\varepsilon_s \cdot m_{s,3} = \frac{\varepsilon_s \cdot m_{s,3}}{l_x{}^2 - 2l_x} \cdot l_x{}^2 - \frac{\varepsilon_s \cdot m_{s,3}}{l_x{}^2 - 2l_x} 2l_x \cdot l_x . \quad (69)$$

Determination of deformations and tensions. After determining the shear bond force, the axial force N_i in each constituent element of the composite bar is determined from the expression:

$$N_i = N_i^0 - T_i + T_{i+1} \quad (70)$$

where T_i, T_{i+1} are the total shear forces in the i[th] bars.

The internal moment M_i in one bar element is equal:

$$M_i = \frac{M^0 E_i I_i}{\sum EI} - \sum_{j=1}^{n} \frac{T_j c_j E_j I_j}{\sum EI} . \quad (71)$$

Here c_i is the distance between centers of gravity of adjacent bars, separated by the i[th] joint; $\sum EI$ is the sum of the stiffnesses of the elements of the composite bar.

The deflection of a bar composed of several elements can be obtained after determining total bending moment:

$$M = M^0 - \sum_{i=1}^{n} T_i c_i M . \quad (72)$$

The axial tensions and linear strains in each element of the composite bar are defined by the expressions:

$$\sigma_{x.i} = \frac{N_i}{A_{ix.i}} + \frac{M_i z_i}{I_i} . \quad (73)$$

$$\varepsilon_{x.i} = \frac{N_i}{E(\lambda) A_i} + \frac{M_i z_i}{E(\lambda) I_i} . \quad (74)$$

Here z_i is the distance from the center of gravity of the cross-section of the i[th] element of the composite bar to the fiber in question.

Then the expression for the curvature of the composite bar is obtained:

$$\frac{1}{r_x} = y'' = -\frac{M}{\sum EI} . \quad (75)$$

The axial force diagram of the composite bar is graduated in all the bars.

Consider the equilibrium of a prism of length dx and height $z_i + b_{i+1}$ cut from the i[th] bar to determine the tangential tensions τ_{zx} in the i^{th} bar. Project all forces onto the x-axis, we obtain:

$$B(z_i)\tau_{zx} = -\frac{d}{dx}\int_{-b_{i-1}}^{z_i} B(t)\cdot\sigma_x(t)dt + \tau_{i-1}, \qquad (76)$$

and taking into account the formula we get (73):

$$\tau_{zx} = \frac{1}{B(z_i)}\left[-\frac{N_i'}{F_i}F(z_i) - \frac{M_i' S(z_i)}{I_i} + \tau_{i-1}\right], \qquad (77)$$

Here $F(z_i)$, $S(z_i)$ are the cross-sectional area of the i[th] bar above the level z_i and the static moment of this area in relation to the central axis of the i[th] bar; $B(z_i)$ is the section width at the level of z_i; for a rectangular cross-section $B(t) = B = const$ and $S(z_i) = S$.

Then taking into account formulas (70) and (71), tangential tensions in each constituent bar at $N_i^0 = const$, are equal:

$$\tau_{zx} = \frac{1}{B(z_i)}\left[\frac{\tau_i - \tau_{i-1}}{F_i}F(z_i) - \frac{Q^0 E_i I_i}{\sum EI}\cdot\frac{S(z_i)}{I_i} + \sum_{j=1}^{n}\tau_i c_i \frac{E_i I_i}{\sum EI}\frac{S(z_i)}{I_i} + \tau_{i-1}\right], \qquad (78)$$

where $Q^0 = M^{0\prime}$.

The angular strains in each element of the composite bar are [19-22]:

$$\gamma_{zx} = \frac{\tau_{zx}}{G(\lambda)} = \frac{\tau_{zx}\cdot 2(1+\nu(\lambda))}{E(\lambda)}. \qquad (79)$$

The angular strains in each joint of the composite bar are:

$$\gamma_{zx,stitch,b,i} = \frac{T'_{zx,stitch,b,i}}{G_m} = \frac{\tau_{zx,stitch,b,i}}{G_m} = \frac{\tau_{zx,stitch,b,i}}{k_{i,m}\cdot\xi_m}. \qquad (80)$$

Find here the value $T'_{zx,stitch,b,i}$ from the system of differential equations (63), i.e., find the value of the tangential tensions $\tau_{zx,stitch,b,i}$.

Then determine the additional force taking into account the "dowel" effect in the stirrups. The force in the stirrups from the axial force q_{sw} for the block that is wedge and arch models is taken into account separately [10, 24-26]:

$$Q_s = \frac{\sigma_s A_s ctg\theta}{\eta_\tau} \le Q_{s,exp}, \qquad (81)$$

$$\Delta_s = \frac{0.5a_{crc,s}\, sin\theta - N_s B_s}{0.5cos\theta} \le \Delta_{exp,s}. \qquad (82)$$

Here η_τ is the transient coefficient before yield strength in the reinforcement $\eta_\tau = 13-17$ (in average $\eta_\tau = 16$), and after yield strength is reached, at $\sigma_s = R_s$, $\eta_\tau = 20-25$ [10, 23-28]; $Q_{s,exp}$ is experimental determined "dowel" force in the reinforcement.

The shear in the crack τ Δ_{crc} is found by the equation (67).

Tangential tensions and angular strains from the "dowel" effect are determined by the expressions:

$$\tau_{zx,stitch,s,i} = \frac{Q_s}{A_s} = \frac{\sigma_s A_s ctg\theta}{\eta_\tau A_s} = \frac{\sigma_s ctg\theta}{\eta_\tau}, \qquad (83)$$

and

$$\gamma_{zx,stitch,s,i} = \frac{Q_s}{A_s G_m} = \frac{\tau_{zx,stitch,i}}{G_m} = \frac{\tau_{zx,stitch,i}}{k_{i,m} \cdot \xi_m} = \frac{\sigma_s ctg\theta}{\eta_\tau \cdot k_{i,m} \cdot \xi_m}. \qquad (84)$$

The total angular strains of concrete and reinforcement are determined by the equation:

$$\gamma_{zx,stitch,sum,i} = \gamma_{zx,stitch,b,i} + \gamma_{zx,stitch,s,i}. \qquad (85)$$

The jumps in the axial tension diagram in each shear joint are determined by the relation T'_i / ξ_i.

Conclusions

Based on these studies, the following conclusions can be drawn.

1. A scheme is proposed for approximating rectangular cross-sections of complexly stressed reinforced concrete structures using small squares to determine the elements a_{pq} of the matrix of stiffness characteristics from $D_{11,**}$ to $D_{33,**}$, and also built direct (for internal forces) and reverse, (where, $\frac{1}{r_x}$, ε_0 and Δ_Q are known, but unknown M_x, N, Q) transition to determine the coefficients - stiffness characteristics of the matrix D_{pq} (p, q - 1, 2, 3) of the compressed area of concrete and tensioned working reinforcement in the system of equations, - static, geometric and physical equations.

2. To solve the problems of determining the design parameters of the limit states of group II in the zone of inclined (cross) cracks, a deformation model of single composite strips was constructed that simulates deformations in the zone of inclined cracks, taking into account the change in the position of the curved axis during the formation of cracks.

3. Rigidity of sections in the zone of inclined cracks of reinforced concrete structures under the action of a bending moment M_{bend}, axial N and shear forces Q for the model of single composite strips are found based on axial strains $\varepsilon_{0,x,j,i,*}$ (from the neutral axis), curvature $\frac{1}{r_{x,j,i,*}}$, as well as shear drifts $\Delta_{Q,j,i,*}$, taking the scheme of breaking down cross sections into small squares and taking into account elastic-plastic strains in compressed concrete and working reinforcement.

4. Based on the proposed hypothesis about the nature of the distribution of strains in a complexly stressed reinforced concrete element with cracks, the conditional shear modulus ξ_m in a single zone of a shear joint was obtained for average relative linear and angular strains from mutual displacements of concrete and reinforcement at the point of the joint adjacent to the shear joint between adjacent cracks. The use of this hypothesis allows us to reduce the order of the system of differential equations of composite A. P. Rzhanitsyn and take into account the presence of cracks in the structure.

5. For shears and internal forces in the model of reinforced concrete composite bars, axial stresses and linear strains in each component bar are obtained. The plot of axial and tangential stresses (strains) in the cross section of a composite bar is stepped.

6. The gradient of the dependence of the crack shear on the diagram of axial and shear stresses in each composite bar for each weld is determined by the ratio of the derivative of the shear force in the i^{th} joint to the conditional shear modulus.

7. The dowel force in the reinforcement Q_S from the shear force in the inclined section was obtained on the basis of the model of the support zone in the form of composite strips and the experimentally determined shear value in the crack.

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